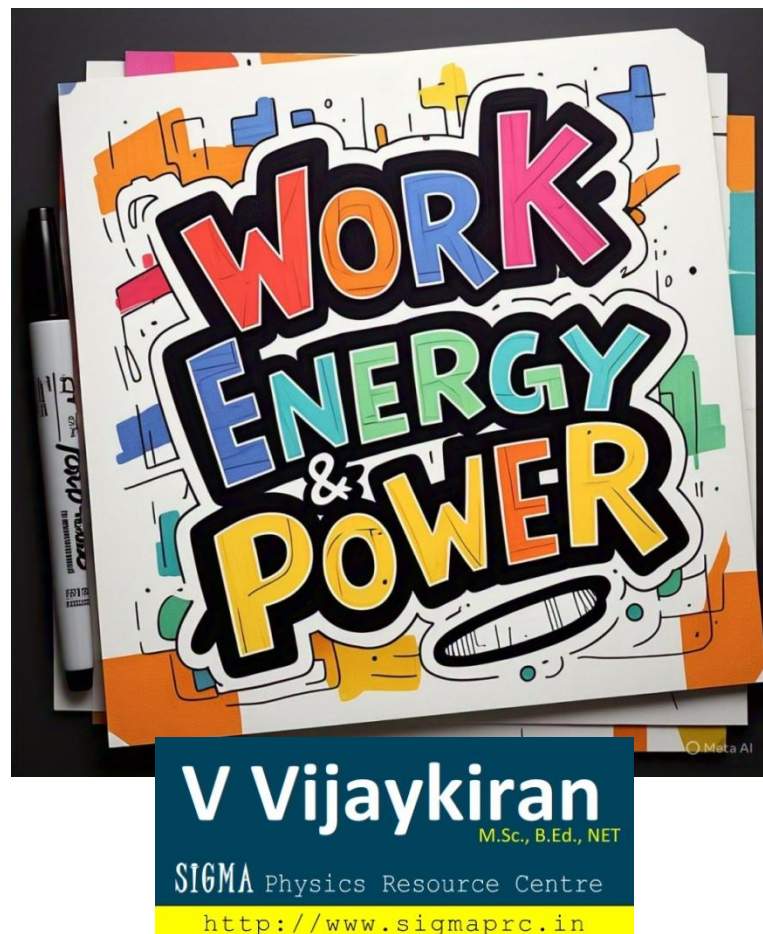


Work, Energy and Power



Note : The notes given in this file is no substitute to the much detailed discussion held in the online/contact classes with active participation of students. It , at best, serves the purpose of ready reference for important concepts/derivations covered in the classes.

Scalar product or dot product of vectors

Scalar product of two vectors is defined as the product of magnitudes of the two vectors and cosine of angle between the vectors.

$$\mathbf{A} \cdot \mathbf{B} = |\mathbf{A}| |\mathbf{B}| \cos(\theta)$$

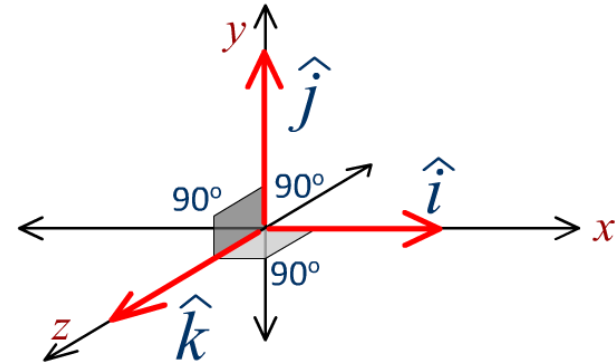
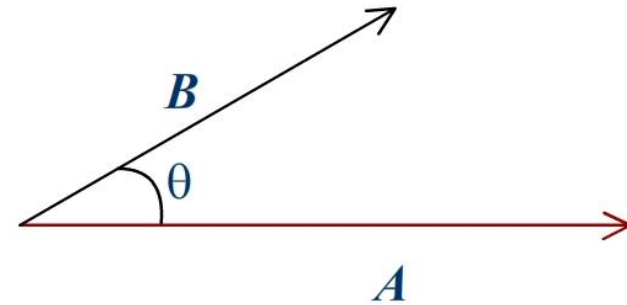
If the vectors are given in their component forms such as

$$\mathbf{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

$$\mathbf{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$$

then the dot product is given by

$$\mathbf{A} \cdot \mathbf{B} = (A_x B_x + A_y B_y + A_z B_z)$$



Properties of scalar product

- ☐ Dot product of two parallel vectors is equal to the product of the magnitudes of the vectors
- ☐ Dot product of two mutually perpendicular vectors is zero
- ☐ Dot product is commutative in nature $\mathbf{A} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{A}$
- ☐ Dot product is distributive $\mathbf{A} \cdot (\mathbf{B} + \mathbf{C}) = \mathbf{A} \cdot \mathbf{B} + \mathbf{A} \cdot \mathbf{C}$
- ☐ $\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0$
- ☐ $\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$
- ☐ For a real number (a constant) λ we get $\mathbf{A} \cdot (\lambda \mathbf{B}) = \lambda \mathbf{A} \cdot \mathbf{B}$

Angle between two vectors

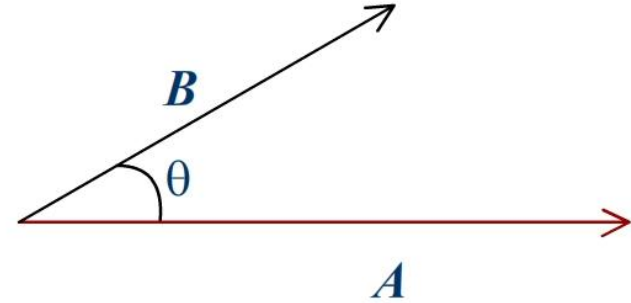
Using the definition of dot product of two vectors, the angle between them is obtained as

$$\mathbf{A} \cdot \mathbf{B} = |\mathbf{A}| |\mathbf{B}| \cos(\theta) \quad \text{--- i}$$

$$\mathbf{A} \cdot \mathbf{B} = (A_x B_x + A_y B_y + A_z B_z) \quad \text{--- ii}$$

$$|\mathbf{A}| |\mathbf{B}| \cos(\theta) = (A_x B_x + A_y B_y + A_z B_z)$$

$$\cos(\theta) = \frac{(A_x B_x + A_y B_y + A_z B_z)}{|\mathbf{A}| |\mathbf{B}|}$$



Assignment on dot product of vectors

Work

Work done by a force acting on a body is defined as the product of component of force in the direction of displacement and the magnitude of displacement of the body.

$$dW = \mathbf{F} \cdot d\mathbf{s}$$

$$dW = |F| |ds| \cos(\theta)$$

$$dW = F_x dx + F_y dy + F_z dz$$

SI unit of work of joule (J)

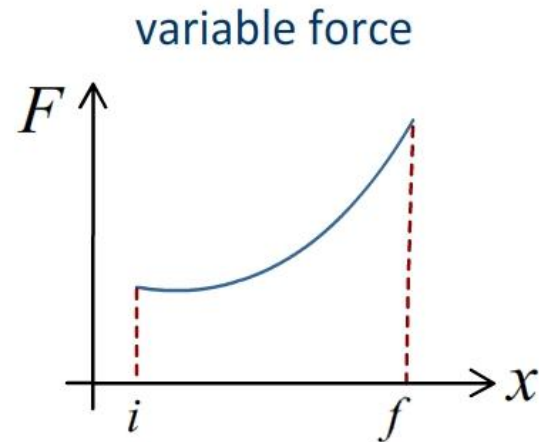
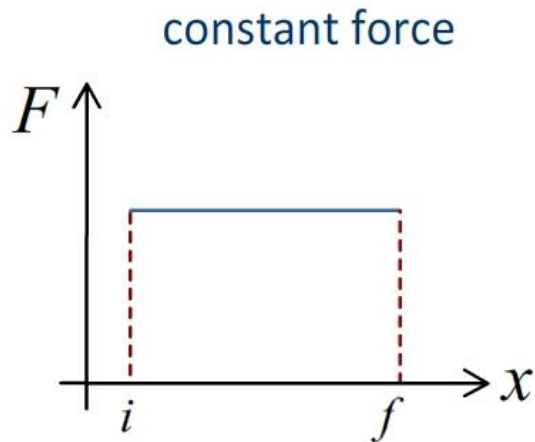
Work done on a body is said to be one joule if 1 newton of force acting on it causes a displacement of 1m in the direction of application of force.

The above expression is for work done by the applied force in causing a small displacement in the body. To calculate the total work done by the force, we must take into consideration all such contributions using suitable mathematical techniques.

Work, Energy and Power

Work done from the force-displacement graph

Work done is given by the area under F - s graph or the area enclosed by F - s graph.



Work, Energy and Power

Calculation of total work using the small contributions

- ❑ Multi step process with constant or linearly varying force in each step :
Calculate work done in each step. Add the work done in all steps (while retaining their $\pm$ signs) to get the total work.
- ❑ Continuous contributions
When the force acting on the body is not uniform (and additionally non-linear) then to calculate the total work done by the force, we must take into consideration all such contributions using suitable mathematical techniques.

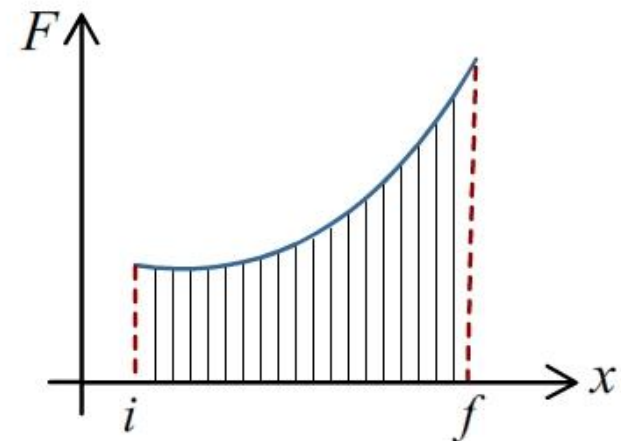


Assignment on dot product of vectors



Mathematical technique of
integration (sneak peek)

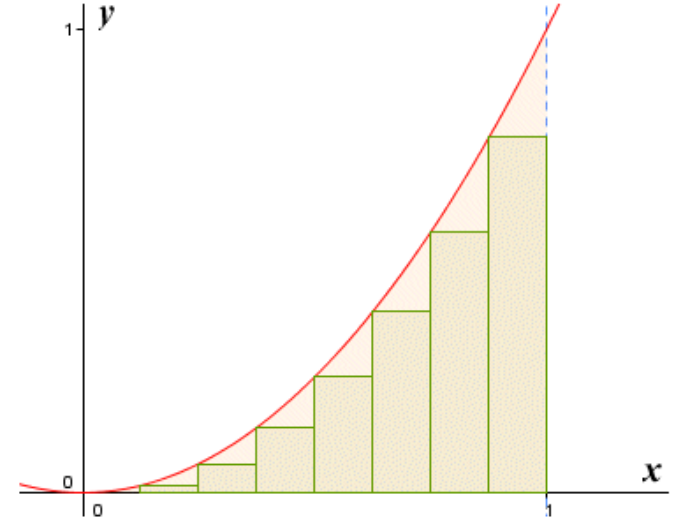
variable force



Work, Energy and Power

A short note on integration

- ❑ It is a process of obtaining the total change as a combination (integration) of very small (and continuous) contributions
- ❑ Integration is used when determination of a quantity involves a product of two or more other quantities which are not constants



For example

A small amount of work (dW) done by a force (F) applied on a body causing a small displacement (dx) is given by

$$dW = F \cdot dx$$

Total work done on the body, as it undergoes a displacement from initial position x_1 to final position x_2 is given by

$$\int dW = \int_{x_1}^{x_2} F \cdot dx$$

Work, Energy and Power

Some important points regarding integration (in the present context)

Indefinite integration

$$\int f(x)dx = F(x) + C$$

Definite integration

$$\int_i^f f(x)dx = [F(x)]_{x_i}^{x_f} = [F(x_f) - F(x_i)]$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$\int dx = x + C$$

$$\int \frac{1}{x} dx = \log_e(x) + C$$

$$\int \sin(\theta) d\theta = -\cos(\theta) + C$$

$$\int \cos(\theta) d\theta = \sin(\theta) + C$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + C$$

$$\int \log_e(x) dx = x[\log_e(x) - 1] + C$$

($\log_e$ is also written as $\ln$)

$$\int Af(x)dx = A \int f(x)dx$$

$$\int (Ax^a + Bx^b) dx = \int Ax^a dx + \int Bx^b dx$$